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The Effect of Ringdyeing on the Color of Fibers

Recently, the authors have proposed a new approach to the prediction of the color of absorbing-scattering substrates such as fabrics.[1] One can expand this treatment to take into account the effect of an inhomogeneity of dye distribution, most easily the condition known as ringdyeing. Garrett and Peters[2] dealt with this problem by expanding Atherton's[3] treatment of the color of fabrics.

Since our approach to this problem is based on the optical properties of the fibers and the medium of observation and on the geometry of the system, it is believed to be potentially more significant than the older treatments.

In our paper, we showed that the pathlength (lp) of light in the fiber is (under the restrictions outlined there)

$$lp = 2\cos\alpha = 2\sqrt{1-(n_1/n_2)^2\sin^2\theta} = (2/m)\sqrt{m^2-d^2} \tag{1}$$

where α is the angle of refraction, θ is the angle of incidence of light, n_1 is the refractive index of the continuous medium, n_2 is that of the fiber, m is the ratio n_2/n_1, and $d = \sin\theta$.

As can be readily seen in Figure 1, the path length in the ring, lpr, is

$$lpr = (2/m)\left[\sqrt{m^2-d^2} - \sqrt{m^2(1-a)^2-d^2}\right] \tag{2}$$

and in the core, lpc is

$$lpc = 2/m\sqrt{m^2(1-a)^2-d^2} \tag{3}$$

where a is the thickness of the core.

If a given amount of dye is distributed uniformily in a cylindrical fiber, its concentration times the coefficient of absorption of the dye (in terms defined in the aforementioned paper) is CK. If this same amount of dye is concentrated in a ring of thickness a, then this quantity, CK_r, becomes

$$CK_r = CK/a(2-a) \tag{4}$$

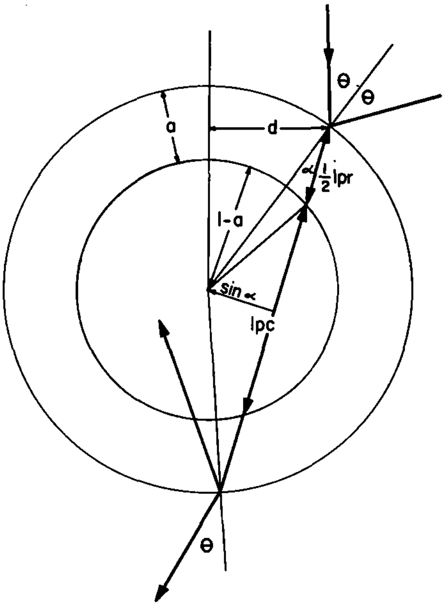

Fig. 1. Cross section of a ringdyed fiber showing a light path through it.

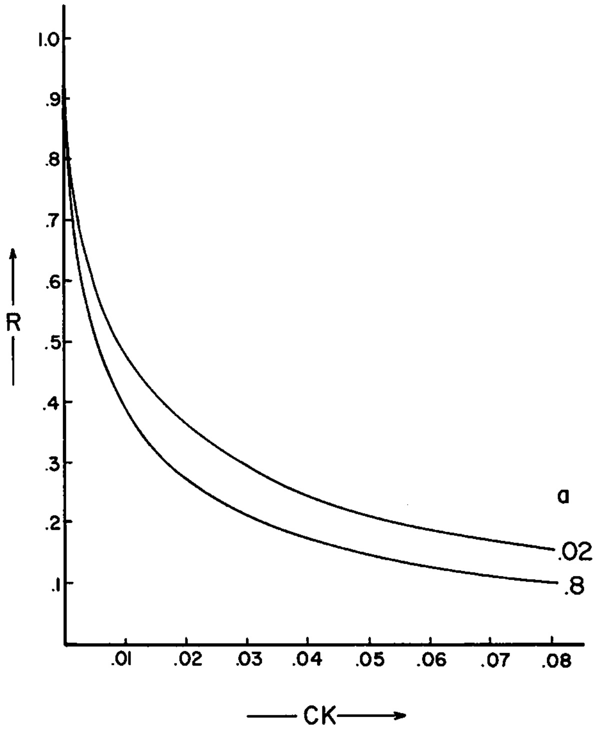

Fig. 2. Reflectance as a function of CK for rings 0.02 and 0.8 radii thick. The plot for $a = 1$ is almost equal to that for $a = 0.8$. Data are for $n_2/n_1 = 1.6$.

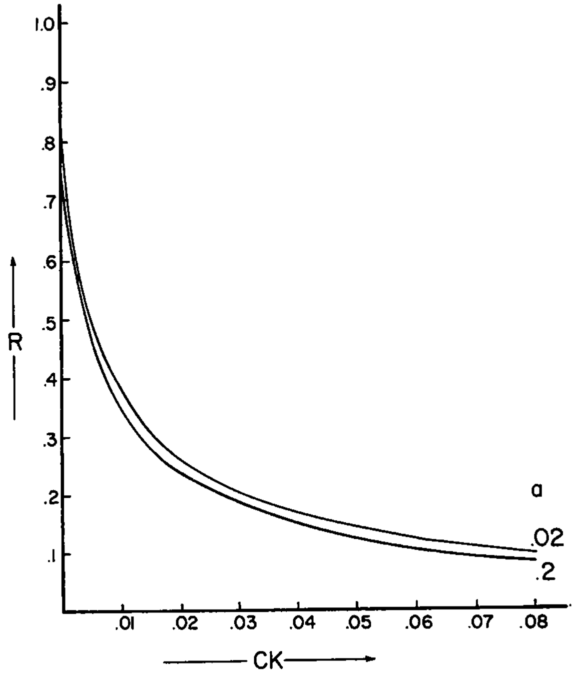

Fig. 3. Reflectance against CK for samples in which the core contains the dye. The radius of the dyed core is $1 - a$. Data are for $n_2/n_1 = 1.6$.

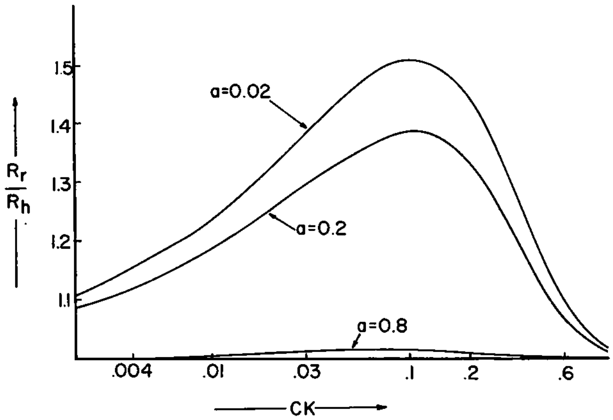

Fig. 4. The ratio of the reflectances of ringdyes to homogeneously dyed fibers plotted against the logarithm of CK. Data are for $n_2/n_1 = 1.6$.

and if it is found in the core, it becomes

$$CK_c = CK/(1 - a)^2. \tag{5}$$

Carrying out the calculations with these new variables, one obtains the relationship between R, the reflectance for different values of the variables CK_r and CK_c at values of a between zero and 1, as shown in the family of curves in Figures 2 and 3. In Figure 4, the relative reflectance ($R_{\text{homogeneous}}/R_{\text{ringdyed}}$) is shown as a function of log CK. Since in a given system the light path and total dye concentration are constant, the various values of CK are proportional to the absorbance of the system. The figure shows what one would, of course, predict qualitatively anyhow, that the effect of ringdyeing is maximum at some intermediate value of absorbance (or reflectance) and that it approaches zero for very large and very small values of CK.

References

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